Units and Measurements

Answer Key & Complete Solutions

Physics Practice Series

Contents

Question 1

Answer:
Plane angle is defined as the ratio of the length of an arc to the radius of the circle in which the arc lies.

$$\theta = \frac{\text{Arc length } (s)}{\text{Radius } (r)}$$

Question 2

Answer:

- **SI unit of energy:** joule (J)
- **Dimensional formula of energy:** $[ML^2T^{-2}]$

Question 3

Answer:
Completed Table:

Name of base quantity	Name of SI unit
Length	metre
Mass	**kilogram**
Time	second
Electric current	**ampere**
Thermodynamic temperature	**kelvin**
Amount of substance	mole
Luminous intensity	candela

Question 4

Answer:

() **Principle of Homogeneity of Dimensions:** According to this principle, physical quantities having the same dimensions can only be added, subtracted, or equated in a physical relation.

() **Derivation for Time Period of a Planet:** Let the time period T depend on radius r, mass M, and gravitational constant G as:

$$T \propto r^a M^b G^c$$

Writing dimensions on both sides:

$$[T] = [L]^a [M]^b [M^{-1} L^3 T^{-2}]^c$$

$$[M^0L^0T^1] = [M^{b-c}L^{a+3c}T^{-2c}]$$

Equating the powers of $M, L,$ and T on both sides:

$$-2c = 1 \implies c = -\frac{1}{2}$$
$$b - c = 0 \implies b = c = -\frac{1}{2}$$
$$a + 3c = 0 \implies a = -3c = \frac{3}{2}$$

Substituting these values back into the equation:

$$T \propto r^{3/2}M^{-1/2}G^{-1/2} \quad \implies \quad T = k\sqrt{\frac{r^3}{GM}}$$

Question 5

Answer:
Completed Table:

Quantity	**SI Unit**	**Symbol of SI unit**
Time	second	**s**
Electric current	**ampere**	A
Solid angle	steradian	Sr
Amount of substance	mole	mol

Question 6 **[2021 IMPROVEMENT]**

Answer:
Let centripetal force F depend on mass m, velocity v, and radius R as:

$$F \propto m^a v^b R^c$$

Substituting the dimensions of each quantity:

$$[MLT^{-2}] = [M]^a[LT^{-1}]^b[L]^c$$

$$[M^1L^1T^{-2}] = [M^aL^{b+c}T^{-b}]$$

Equating powers of $M, L,$ and T on both sides:

$$a = 1$$
$$-b = -2 \implies b = 2$$
$$b + c = 1 \implies 2 + c = 1 \implies c = -1$$

Therefore:

$$F \propto m^1v^2R^{-1} \quad \implies \quad F \propto \frac{mv^2}{R} \quad \text{or} \quad F = k\frac{mv^2}{R}$$

Question 7

Answer:
From Hooke's Law: $F = kx \implies k = \frac{F}{x}$

$$[k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$$

Question 8

[2022 MARCH]

Answer:
Given relation: $v = \sqrt{\frac{GM}{R}}$

- **LHS Dimension:**
$$[v] = [LT^{-1}]$$
- **RHS Dimension:**
$$\left[\sqrt{\frac{GM}{R}}\right] = \sqrt{\frac{[M^{-1}L^3T^{-2}][M]}{[L]}} = \sqrt{[L^2T^{-2}]} = [LT^{-1}]$$

Since LHS $=$ RHS $= [LT^{-1}]$, the given equation is **dimensionally correct**.

Question 9

Answer:
(b) distance (A light year is the distance light travels in one vacuum year: $\approx 9.46 \times 10^{15}$ m).

Question 10

Answer:
(iv) 10^{-10} m

Question 11

[2023 MARCH]

Answer:

() **Principle of Homogeneity of Dimensions:** Only physical quantities having the same dimensions can be added, subtracted, or equated.

() **Dimensional Check of $\frac{1}{2}mv^2 = mgh$:**

$$[\text{LHS}] = \left[\frac{1}{2}mv^2\right] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$
$$[\text{RHS}] = [mgh] = [M][LT^{-2}][L] = [ML^2T^{-2}]$$

Since [LHS] $=$ [RHS], the equation is **dimensionally correct**.

() **Significant Figures:**

(ı*) 0.060607 m^2 $\implies$ **5** significant figures (leading zeros are not significant).

(ı*) 6.0320 N m^{-2} $\implies$ **5** significant figures (trailing zero after decimal point is significant).

Question 12

Answer:
(b) Mass (Mass is one of the seven fundamental SI base quantities).

Question 13 **[2023 IMPROVEMENT]**

Answer:

() Refer to Question 11(b) for the complete proof: $[\text{LHS}] = [\text{RHS}] = [ML^2T^{-2}]$, hence dimensionally correct.

() According to Einstein's special theory of relativity, the term $(1 - v^2)$ must be dimensionless. Since v is velocity, it must be divided by c^2 (where c is the speed of light) to make the term dimensionless.

The correct equation is:

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

Question 14

Answer:
c) $[M^{-1}L^3T^{-2}]$
Derivation: From $F = \frac{Gm_1m_2}{r^2} \implies G = \frac{Fr^2}{m_1m_2}$, so $[G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$.

Question 15

Answer:
Two principal uses of dimensional analysis:

1. To check the correctness of a physical equation.
2. To derive the relation between various physical quantities.

Question 16

[2024 MARCH]

Answer:

(ı*) 6.320 J $\implies$ **4** significant figures.

(ı*) 2370 g cm^{-3} $\implies$ **3** significant figures.

Question 17

Answer:
b) Temperature (Temperature is a fundamental quantity, whereas Velocity, Force, and Density are derived quantities).

Question 18

[2025 MARCH]

Answer:

() **Principle of Homogeneity of Dimensions:** Only physical quantities having the same dimensions can be added, subtracted, or equated.

() **Derivation for Time Period of Simple Pendulum:** Let time period T depend on mass m, length l, and acceleration due to gravity g:

$$T \propto m^a l^b g^c$$

Writing the dimensions of both sides:

$$[T] = [M]^a [L]^b [LT^{-2}]^c$$

$$[M^0 L^0 T^1] = [M^a L^{b+c} T^{-2c}]$$

Comparing powers of $M, L,$ and T on both sides:

$$a = 0$$

$$-2c = 1 \implies c = -\frac{1}{2}$$

$$b + c = 0 \implies b = -c = \frac{1}{2}$$

Substituting powers back into the expression:

$$T \propto m^0 l^{1/2} g^{-1/2} \quad \implies \quad T \propto \sqrt{\frac{l}{g}} \text{ or } T = k\sqrt{\frac{l}{g}}$$